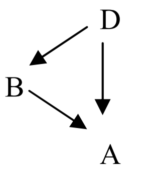
Genetic relationships and close inbreeding - solutions

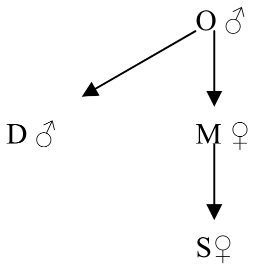
1. a)



$$F_A = \frac{1}{2}a_{BD} = \frac{1}{2}\times\left((\frac{1}{2})^{1+0}(1+F_D)\right) = \frac{1}{2}\times\left(\frac{1}{2}\times(1+0)\right) = \frac{1}{4} = 0.25$$

b) $$a_{AD} = \left((\frac{1}{2})^{2+0} + (\frac{1}{2})^{1+0}\right)(1+F_D) = \left((\frac{1}{2})^{2} + (\frac{1}{2})^{1}\right)(1+0) = \frac{1}{4}+\frac{1}{2} = 0.75$$

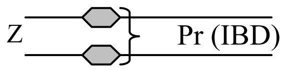
2. a)



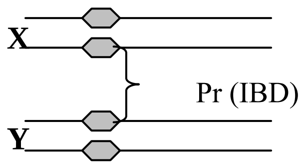
$$a_{DS} = (\frac{1}{2})^{1+2}\left(1+F_O\right) = (\frac{1}{2})^{3} = \frac{1}{8}$$

b) $$F_{progeny} = \frac{1}{2}a_{DS} = \frac{1}{2}\times\frac{1}{8} = \frac{1}{16} = 0.625 = 6.25\%$$

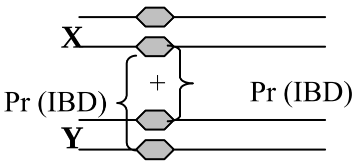
3. a) F_Z = The probability that two alleles in a locus of an individual Z are identical by descent (IBD)



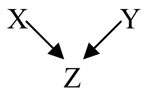
b) f_{xy} = The probability that a randomly sampled gene from individual X is identical by descent with one randomly sampled gene of the same locus from individual Y



c) a_{xy} = The probability that a randomly sampled gene from individual X is identical by descent with any of the genes in the same locus from individual Y



d)



$$F_Z = f_{xy} = \frac{1}{2}\ a_{xy}$$

4. D is inbred: $F_D = \frac{1}{2}\ a_{BA} = \frac{1}{4}$

Three paths must be summarized:

$$a_{EF} = (\frac{1}{2})^{3+2}(1+0) + (\frac{1}{2})^{2+2}(1+0) + (\frac{1}{2})^{1+1}(1+\frac{1}{4})$$

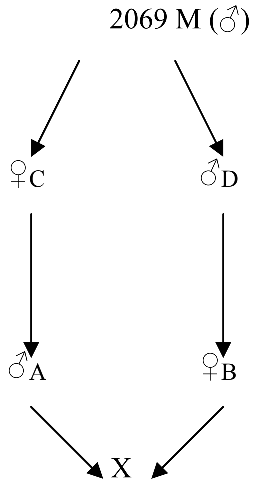
EDBACF EDACF EDF

$$= (\frac{1}{2})^5 + (\frac{1}{2})^4 + (\frac{1}{2})^2 \times \frac{5}{2} = \frac{1}{32} + \frac{1}{16} + \frac{5}{16} = \frac{13}{32}$$

$$F_X = \frac{1}{2}\ a_{EF} = \frac{1}{2} \times \frac{13}{32} = \frac{13}{64} \approx 0.20$$

5.

a)



The inbreeding coefficient of the progeny (F_x) equals half the additive relationship (a_{AB}) between parents A and B

$$a_{AB} = (\frac{1}{2})^{m+n}(1+F_{2069M})$$

where m and n are the number of arrows between A, B and the common ancestor (2069 M)

Thus

$$a_{AB} = (\frac{1}{2})^{2+2}(1+0) = \frac{1}{16} = 0.0625 \quad or \quad 6.25\%.$$

The inbreeding coefficient for X is

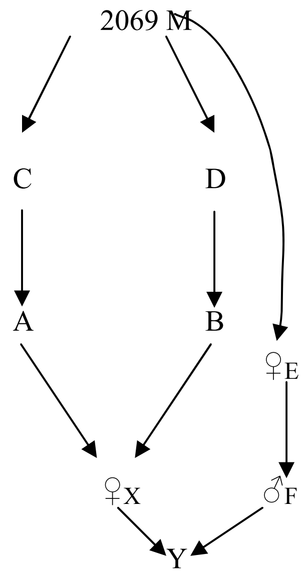
$$F_x = \frac{1}{2} \times a_{AB} = \frac{1}{2} \times \frac{1}{16} \times \frac{1}{32} = 0.03125 \quad or \quad 3.125\%.$$

Here we assumed that the ancestor (2069 M) is not inbred.
If e.g. the inbreeding coefficient for 2069 M is 5%, then

$$a_{AB} = (\frac{1}{2})^{2+2}(1+0.05) = 0.0625 \times 1.05 = 0.066 \quad or \quad 6.6\% \quad and$$

$$F_x = 3.3\%$$

b)



The relationship between the parents of Y is

$$a_{XF} = \left((\frac{1}{2})^{3+2} + (\frac{1}{2})^{3+2}\right) \times (1+F_{2069M})$$

i.e. both paths X, A, C, 2069 M, E, F and
X, B, D, 2069 M, E,F are summarized

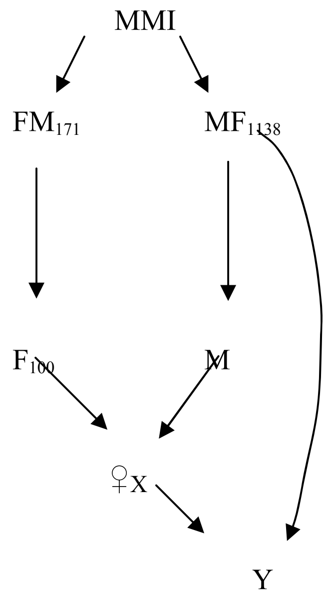
If F_{2069M} is 0 (2069 M not inbred)

$$a_{XF} = \frac{1}{32} + \frac{1}{32} = \frac{1}{16} = 0.0625 = 6.25\%$$

The inbreeding coefficient for Y is then

$$F_Y = \frac{1}{2} \times \frac{1}{16} = \frac{1}{32} = 0.03125 = 3.125\%$$

c)



The heifer X is a grand-daughter to 1138 S and furthermore related to him through 2069 M.
The relationship between them is

$$a_{X-1138S} = (\frac{1}{2})^{2+0}(1+F_{1138S}) + + (\frac{1}{2})^{3+1}(1+F_{2069M}).$$

If 1138 S and 2069 M are not inbred, i.e. F_{1138S} and $F_{2069\ M}$ = 0 then

$$a_{X-1138S} = (\frac{1}{2})^{2+0} + (\frac{1}{2})^{3+1} = \frac{1}{4} + \frac{1}{16} = = \frac{5}{16} = 0.3125 = 31.25\%.$$

The inbreeding coefficient for an offspring, if the heifer X is inseminated by 1138 S would be:

$$F_Y = \frac{1}{2} a_{X-1138S} = \frac{1}{2} \times \frac{5}{16} = \frac{5}{32} = 0.15625 = 15.6\%.$$

d) 15% is definitely too high to be recommended